COMPARING ALTERNATIVES IN THE LAW[1]

Legal applications of Qualitative Comparative Reasoning

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Abstract. This paper argues the thesis that a particular style of reasoning, qualitative comparative reasoning (QCR), plays a role in at least three areas of legal reasoning that are central in AI and law research, namely legal theory construction, case-based reasoning in the form of case comparison, and legal proof. The paper gives an informal exposition of one particular way to deal with QCR, based on the author's previous work on reason-based logic (RBL). Then it contains a substantially adapted formalisation of RBL, to make RBL suitable for dealing with QCR. The paper concludes with a brief discussion of related work.

Key words: qualitative comparative reasoning, reason-based logic, theory construction, case-based reasoning, legal proof

1. Introduction

Theory construction, case comparison and argumentation about evidence are topics that have received considerable attention in research on AI and law (e.g. Ashley 1991, Hage 2001, Bench-Capon and Sartor 2003, Hage and Sartor 2003, Roth 2003, Prakken e.a. 2003, and Bex e.a. 2003). Apparently these issues are relatively unrelated. In this paper I will argue that they share at least a particular style of reasoning which has to my knowledge not been developed explicitly before, namely qualitative comparative reasoning (QCR). In the first part of this paper, I will introduce a particular style of QCR by means of a simple non-legal example and explain informally the logical mechanisms at work. Then I will argue that this style of QCR plays (or can play) a role in the three mentioned research areas. In the second part of the paper, QCR will be formalized by means of Reason-based Logic.

A. THEORY

2. Right and better

When your old car has broken down and you must decide which brand your new one should be, your main problem will probably not be of a logical nature. Nevertheless, if you have to make a choice between for instance a Mercedes, a Volvo, and a Porsche, the logic underlying the decision is interesting. Each brand of car has advantages and disadvantages and rational decision making requires a form of balancing these (dis)advantages. The easiest case would be if there were a common scale against which all brands could be measured, because then the only 'logic' involved would be to pick the brand with the highest score. However, when this method is not available, other ways to rationalize the decision making process must be looked for.

Another way to deal with this kind of issue is to transform it into the issue whether one should buy a particular brand of car, for instance a Volvo. Logically this would boil down to balancing the reasons for and against buying a Volvo. It is well imaginable that if the issue is framed this way, the reasons for buying a Volvo outweigh the reasons against buying one. It is, however, equally well imaginable that in a similar way the reasons for buying a Mercedes outweigh the reasons against buying one, and that the reasons for buying a Porsche outweigh the reasons against buying a Porsche. If the decisions are taken as independent of each other, one might well end up with buying three cars! What we need

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to know is not merely whether it is right to buy a particular brand of car, but whether it is better to buy a Mercedes, a Volvo or a Porsche.

Obviously the decisions are not independent of each other, and one way to take this into account is to treat every reason to buy a Porsche as a reason against buying a Mercedes and against buying a Volvo. On this approach, the mutual dependence between the decisions is taken into account, and a decision to buy one particular brand of car is implicitly a decision not to buy a car of one of the other brands.[3]

This is a viable way to deal with the issue, as long as the number of alternatives is limited. If the number of alternatives is large, the situation becomes problematic, because every reason to buy a particular brand of car becomes a reason against buying a car of any other brand. Apart from the complexity this leads to, it is unrealistic, because some reasons to buy a particular brand of car will also be reasons pleading for other brands. For instance, one reason to buy a Mercedes is that it is a safe car. This would also be a reason to buy a Volvo. Another reason to buy a Volvo might be that it fits with the image that one wants to create. This same reason might also plead for buying a Porsche. A reason against buying a Volvo would be that it is less suitable for car racing, and this reason pleads also against buying a Mercedes. And so on …

A more attractive way would be to collect for each brand of car the reasons pleading for it and the reasons pleading against it. The first step would then be to decide for every brand whether it is a viable choice. This step can be taken by balancing the reasons for and against buying a car of this brand. If the reasons against buying a car of a particular brand outweigh the reasons for buying it, cars of this brand can be ignored. In the second step, the remaining brands should be compared. If brand A is in some respects better than brand B, and in no respect worse, brand A is preferable to brand B and brand B can be disregarded. It is well possible that this process of elimination leaves only one alternative over, and then the decision can be taken purely on basis of qualitative reasoning. If more than one alternative remains, additional decision making is necessary.

3. Qualitative comparative reasoning

Suppose that one must choose between buying a Volvo and buying a Mercedes. A Volvo has two reasons pleading for it, namely that it is a safe car, and that there is a Volvo dealer next door. It has the disadvantage that it is an expansive car. A Mercedes is also expensive, but has (in the example) only one advantage, namely that it is a safe car. There happens to be no Mercedes dealer in the neighbourhood. Under these circumstances, everything that pleads for a Mercedes also pleads for a Volvo, but a Volvo has an additional reason pleading for it, namely the availability of a dealer nearby. Moreover, a Volvo and a Mercedes have the same reason pleading against it, namely that they are expensive. It seems, therefore, that a Volvo is preferable to a Mercedes. This is a reasonable conclusion, even in the absence of any information concerning the (relative) weights of the reasons that the cars are safe, that there is a Volvo dealer nearby, and that the cars are expensive.

Analogously it is reasonable to conclude that a Mercedes is preferable to a Porsche, if a Mercedes and a Porsche have the pro-reason in common that they are German cars (for those who like German cars), and they also share the con-reason that they are expensive, while a Porsche has the additional disadvantage that it liable to be stolen.

3.1. Comparing alternatives and reason sets

In general alternative A is preferable to alternative B if either:
the set of reasons pleading for A is ‘stronger’ than the set of reasons pleading for B, while the set of reasons pleading against A is not ‘stronger’ than the set of reasons pleading against B; or

1. the set of reasons pleading against B is ‘stronger’ than the set of reasons pleading against A,
2. while the set of reasons pleading for B is not ‘stronger’ than the set of reasons pleading for A; or
3. both 1 and 2 hold.

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3 This approach is most explicitly taken in Brewka and Gordon 1994. See also the discussion of Gordon and Karacapilidis 1997 in section 14.

Until now the examples dealt with the qualitative comparison of *alternatives* in terms of reasons pleading for and against them. It is also possible to apply qualitative comparative reasoning to *sets of reasons*. These sets can be compared qualitatively with regard to their relative 'strength'.

In the above characterisation of when one alternative is preferable to (better than) another alternative, I placed the word 'stronger' between quotes, because the notion of strength needs to be elaborated. In the examples I implicitly assumed that supersets were 'stronger' than their subsets, but intuitive as this may be at first sight, it ignores that individual reasons have a dimension of weight, and that this dimension may interfere with the sheer number of reasons. For instance, if a Volvo is much more expensive than a Mercedes, its additional expensiveness might be more important than the presence of a dealer nearby, with as consequence that a Mercedes might be preferable to a Volvo, even though a Volvo has more reasons pleading for it.

Moreover, the suggestion that the same reason can plead for or against several alternatives is somewhat misleading. It may seem that their safety is a reason that pleads both for buying a Volvo and a Mercedes, but on closer examination the concrete reason for buying a Mercedes is that *a Mercedes* is a safe car, while the reason for buying a Volvo is that *a Volvo* is a safe car. Buying a Mercedes and buying a Volvo share the abstract pro-reason 'being a safe car', but they do not share concrete reasons. However, the actual reasons for buying these cars are the concrete reasons, not the abstract ones. I will deal with this issue in terms of 'similar reasons', concrete reasons that instantiate the same abstract reason. For instance, the reason that a Volvo is a safe car is similar to the reason that a Mercedes is a safe car.

The issue of weights has also to do with this distinction between abstract and concrete reasons, because the weights of reasons are attached to concrete reasons. This means that the concrete reason that a Mercedes is a safe car may have a different weight than the concrete reason that a Volvo is a safe car. Arguably abstract reasons have a dimension of weight too, and concrete reasons inherit this weight 'by default'. In this case, the reason that a Mercedes is a safe car would by default have the same weight as the reason that a Volvo is safe car. But even if this line of argument were correct, it may be the case that similar reasons in a concrete case have different weights and that this interferes with the number of reasons pleading for and against alternatives. Only if the weights of the similar reasons are identical, the strengths of two sets of reasons can be compared qualitatively by means of the numbers of their elements.

The last point can also be turned around: if two sets of reasons have similar elements, their relative strengths can be compared on the basis of the weights of their elements. For instance, if both a Volvo and a Mercedes have one reason pleading for them, namely that they are safe cars, the relative strength of these unitary sets is determined by the weights of these reasons. For instance, if a Mercedes is safer than a Volvo, the weight of the reason that a Mercedes is a safe car is by default bigger than the weight of the reason that a Volvo is a safe car. In that case the set of reasons consisting of the reason that a Mercedes is a safe car is 'stronger' than the set of reasons consisting of the reason that a Volvo is a safe car.

3.2. Degrees and probabilities

The same example also illustrates a different point, concerning the relation between the 'degree' in which a fact obtains, and the weight of the reason that this fact constitutes. Let me be more concrete. A Mercedes is not just safe or not safe, but it is safe to a certain degree. In a similar way it is expensive in a certain degree. Some kind of facts – one might call them 'dimensions'[4] – do not just obtain or not obtain, but they obtain in degrees. If such facts are concrete reasons, the weights of these reasons will usually depend on the degree in which these facts obtain. If car A is more expensive than car B, which is also expensive, the fact that car A is expensive is a stronger reason against buying car A than the fact that car B is expensive is a reason against buying car B.

A similar phenomenon occurs in connection with probabilities. Reasons pleading for and against alternatives, especially when these alternatives are lines of action, often will concern the consequences of adopting the alternatives. These consequences have a certain degree of probability and an attractive consequence will lead to a stronger pro-reason, if the probability of the consequences is higher.

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4 This is the term used by Ashley 1991. See also Bench-Capon and Rissland 2001, about the relevance of these dimensions.

Similarly, an unattractive consequence will lead to a stronger con-reason if the probability of the bad consequences is higher.

As these examples illustrate, the dimension of weight of reasons can be used to reflect two other dimensions of reasons, namely the degree in which the reason-giving facts obtain and the probability of the consequences.

3.3. The 'logic' of reason comparison

If two 'similar' sets (sets that contain similar reasons) have more than one element, they can only be compared qualitatively on basis of the weights of their reasons if all the differences in weight work in the same direction. Suppose, for instance, that a Volvo and a Porsche have the same pro-reasons, namely their social status and their suitability for holiday purposes. If a Volvo is both better for status and for holiday purposes, the set of pro-reasons for a Volvo is by default stronger than the set of pro-reasons for a Porsche. But if a Volvo is better for holiday purposes, but a Porsche better for social status, the sets of reasons cannot be compared qualitatively, at least not without additional information.[5]

COMPARISON OF ALTERNATIVES

The above can be generalized as follows. Each alternative in a set of alternatives has one (possibly empty) set of reasons pleading for it (the pro-reasons) and a similar set of con-reasons. Two alternatives can be compared by pair wise comparing the sets of pro- and con-reasons. For this purpose the relations stronger, weaker and equal are used. A set of reasons can be stronger than, weaker than, or equal to another set. These three relations are mutually exclusive. They are not exhaustive, however, because in some cases sets of reasons cannot be compared qualitatively.

Given these relations between sets of reasons, it is sometimes possible to establish on logical grounds which of two alternatives, if any, is preferable to the other. If the two alternatives are called A and B, and the relevant sets of reasons pleading for and against A and B are called Pro-A, pro-B, con-A and con-B, it holds that:

Alternative A should be preferred to (is better than) alternative B if:

- Pro-A is stronger than pro-B, and con-B is either equal to or stronger than con-A; or
- Pro-A is equal to pro-B, and con-B is stronger than con-A.

Alternative A is equal to alternative B if pro-A is equal to pro-B, and con-A is equal to con-B.

It is not possible to establish *on the above mentioned grounds* which alternative is better than the other, or whether the alternatives are equal to each other if either

- Pro-A is stronger than Pro-B, while con-a is stronger than Con-B, or
- Pro-A is weaker than Pro-B, while Con-a is weaker than Con-B, or
- Pro-A and Pro-B, or Con-A and Con-B cannot be compared qualitatively

COMPARISON OF REASON SETS

Sometimes it is possible to determine on logical grounds whether a set of reasons is stronger, or weaker than, or equal to another sentence. In this connection two aspects of these sets are taken into account, namely whether one set is a proper (similar-)superset of the other, or - in other words - whether one set contains all similar elements of the other and then some more, and whether one or more of the reasons if one of the sets weigh more than the similar reasons in the other set.[6]

A set is stronger than another set (and the other set is weaker) if:

- it is a proper (similar-) superset of the other, and none of its reasons weighs less than the similar reason in the other set (if there is such a similar reason); or

5 But see section 3.4.

6 As described above, the degree in which a reason (a dimension) obtains, is taken into account through the weight of the reason.

- all its elements are similar to elements of the other set, and the other way round, and none of its reasons weighs less than the similar reason in the other set, and at least one of its reasons weighs more than the similar reason in the other set.

A set is equal to another set if:

- all its elements are similar to elements of the other set, and the other way round, and
- all of its reasons have the same weight as the similar reason in the other set.

3.4. Weak Transitivity

Often two sets of alternatives will not be comparable on logical grounds alone. Then additional decision making is necessary to establish which one is better. For instance, if a Volvo is a safer car than a Porsche, but a Porsche is better for one's social status, and these are the only relevant reasons, it is not possible to establish on logical grounds which brand is better. It seems therefore, that the theory of qualitative comparative reasoning described here has a limited scope of application.

Although there are certainly limitations, matters are not as bad as may seem at first sight, however. This has to do with the weak transitivity of the better than-relation. Suppose that a decision has been made that a Volvo is better than a Porsche. Suppose, moreover, that a Mercedes is just as safe as a Volvo and is also better for one's social status, and there are no other relevant reasons, then it is possible to determine on logical grounds that a Mercedes is better than a Volvo. But then, since it has been established by decision making that a Volvo is preferable to a Porsche, it seems rational to assume that a Mercedes must, in the absence of special circumstances, be better than a Porsche too.

This can be generalized as follows: If alternative A is better than alternative B, and if C is better than A, then, *by default*, C will be better than B too. Another way to say this is that the better than relation is *weakly transitive*. The weakness of the transitivity consists in the defeasibility of the application of transitivity.

Weak transitivity does not only hold for the better than relation as applied to alternatives, but also for the stronger than, weaker than, and equal to relations as they hold between sets of reasons. For instance, if a Mercedes and a Volvo are both reliable and save cars, while the Volvo is safer, but the Mercedes is more reliable, the sets of pro-reasons for a Mercedes and a Volvo cannot be compared purely on logical grounds. If it is decided that the pro-reasons for a Mercedes are stronger than those for a Volvo, and if it is somehow (maybe on purely logical grounds) established that the pro-reasons for a Lexus are stronger than those for a Mercedes, it can by default be derived that the pro-reasons for a Lexus are stronger than those for a Volvo.

The theory of Qualitative Comparative Reasoning (QCR) formulated above is formalized and made more precise in the sections 11f. But first I will illustrate how QCR can be used in three important fields of legal reasoning, namely those of legal theory construction, of case based reasoning, and of legal proof.

4. Theory construction

In a series of papers[7], culminating in his book *Law's Empire*, Dworkin has developed an intuitively attractive picture of legal theory construction. This picture recognizes three stages in constructing the law (Dworkin 1986, p. 65f). The first stage, the so-called pre-interpretative stage, consists of a preliminary identification of the rules, standards, and (generalized) decisions that make up the law. In this connection one might think of an inventory of the rules and standards that can be found in statutes, cases, and doctrinal literature. The second, interpretative, stage consists of an identification of the principles (including values and policies) that underlie (in the sense of explain), or are part of the legal phenomena identified in the first stage. The rules etc. identified in the first stage are to be seen as means to realize the principles identified in the second stage, but they are not necessarily the best way to realize them. The purpose of the third, reforming, stage is to formulate (relevant parts of) the set of rules, including (generalized) decisions of cases, that best realizes the principles identified in the second stage.

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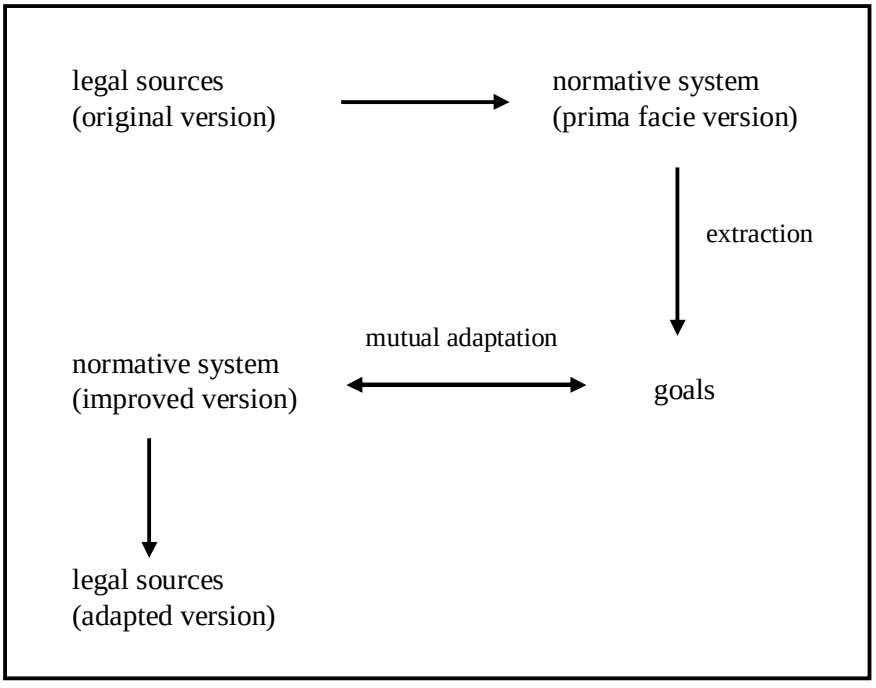
[7] The papers in question are in particular the paper 'Hard Cases', included in Dworkin 1978, and the papers in part two (Law as interpretation) of Dworkin 1985.

Abstracting a little from Dworkin's analysis, it is possible to distinguish within a theory of the law three subsets of elements. The first subset consists of the sources of the law, with a prominent place for pieces of legislation and for individual cases as decided by the judiciary. The second subset consists of the principles, policies, rights and values that underlie and form the inspiration for the law. And finally there is the law as a set of generic cases, with the legal consequences attached to them by the law. Hence fort I will call the first subset the *legal sources*[8], the second subset the *legal goals*[9], and the third set the *normative system*[10].

In the process of legal theory construction, the legal sources determine a rough first account of the normative system based upon them. This prima facie normative system forms so to speak the set of data that the theory must explain. As in empirical theories, it is possible that some of the data must be disregarded if they do not fit in the best theory that can be constructed from them. By means of inductive and abductive reasoning, a set of goals can be identified as underlying the prima facie version of the normative system. Given these goals it is possible to devise a normative system that realizes them best, and given such an ideal normative system, it is possible to devise an adapted set of sources (new legislation or decisions in upcoming cases) by means of which such an ideal normative system is realized.

Ideally, the new sources should deviate as little as possible from the actual sources that function as input to the theory[11], because a good theory of the law should be a theory of the *law as it actually is*, and not merely a theory of ideal law. Yet, the goals underlying the law are also part of the law, and the normative system should also reflect them. Obviously there is a certain tension here, and it depends on ones legal-philosophical disposition how the balance between the actual sources and the legal system that is ideal in the light of the principles underlying the sources is struck.

Graphically the development of the relations between the three subsets of a legal theory can be depicted as follows:



[8] Legal sources in the sense intended here are not statutes or case law in general, but individual regulations and individual cases.

[9] Notice that these goals include a broad spectrum of legally relevant entities, such as (human) rights, legal principles, and policies. For the present purposes these different entities are all lumped together.

[10] The use of the expression 'normative system' for generic cases with their legal consequences was inspired by the use of this expression in Alchourrón and Bulygin 1971.

[11] Where case law is concerned, it is not even possible to change the existing body of case law, as it is possible to change legislation. Old case law can at most be disregarded as outdated by new case law or legislation.

5. Comparing solutions for a case type

One aspect of the mutual adaptation of sources, principles and normative system is the determination of the ideal normative system, given a particular set of goals. Let us have a look at an extended example that illustrates the reasoning about whether a particular solution for a type of case should be part of the theory (should rationally be accepted) given the set of goals that are included in the theory in question. The example consists of variations on the so-called Lebach-case, which was made familiar by Alexy (1979 and 1996).

The standard case runs as follows: A person, let us call him E, who was condemned for abduction and subsequent murder of his victim, is released from prison after ten years. A tabloid journal jumps on this news and uses the occasion to publish an article on the dangers of abduction in general. The article is illustrated with a photograph of E just after his release. E attempts to prevent circulation of the journal.

The judge who must decide on this case should balance two goals. One goal is the freedom of the press; the other one is the goal that one should respect other persons' privacy. Let us assume that the judge decides that in cases like this, privacy protection outweighs freedom of the press.

Now let us change the case a little bit by adding the fact that the news that E was to be released was given to the press on the condition that no photographs would be taken. This forms an additional reason against publication of the article, because the effects of the offence are even enlarged by publishing the photo that was illegally taken. As a consequence the decision that, ceteris paribus, it is not allowed to publish photographs of recently released prisoners if the potential publisher undertook the obligation not to take photographs at all, has even more support in the goals of the theory than the first decision, and is in this sense better.

It is possible to think of another change in the case which leads to a different conclusion. As yet, the question whether the released prisoner objects to the publication has not been taken into consideration yet. It was tacitly assumed that he did object, but this need not be the case, in particular not if he were to be compensated financially for the publication. A solution to the effect that publication is only allowed with explicit consent of the person concerned would take a new goal into consideration, namely personal autonomy. This solution would have the pro- and the con-reasons of the first one, presumably with the same weights, but would have an additional pro-reason because it is supported by the goal of autonomy. As a consequence, the last solution is better than the first one.

A similar argument can be made for the case that a potential decision has similar reasons that plead for it as the old decision, but that the reasons that plead against it are a strict subset of the reasons pleading against the old decision. This would be illustrated by the case in which the tabloid journal has contracted with E that no publication of his photograph would be made. It is arguable that the freedom of the press is not infringed by a prohibition that was voluntarily undertaken by the journal. Since the freedom of the press was a reason against the prohibition, the balance of reasons is moved towards the prohibition if this con-reason is taken away. As a consequence the solution that publication is prohibited if the potential publisher has voluntarily undertaken the obligation not to publish, has even stronger support than the original prohibition.

It is possible that a set of reasons is strengthened by adding new reasons to it, but also by strengthening the reasons that occur in it. This is illustrated by the solution that not only forbids publication of the photograph, but also prescribes that the photograph is destroyed. This solution provides better protection of privacy, and is therefore better than the simple prohibition.[12]

A similar argument can be made for the case that one or more of the reasons pleading against the new decision are weaker than the corresponding reasons pleading for the old decision. For instance, a solution that allows photographs, as long as the persons on the photographs are not recognisable, makes a smaller infringement on the freedom of the press, while the protection of the privacy remains the same. Such a solution would therefore be better than a mere prohibition of publishing photographs.

The findings from the discussed examples can be summarized in the following global guidelines for the comparison of possible solutions for a type of case:

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[12] Arguable this solution would infringe the property right of the journal, but for the sake of the example, this complication is ignored.

- A solution for a case that promotes a goal should pro tanto be adopted.
- A solution for a case that detracts from a goal should pro tanto be rejected.
- If a solution for a case promotes the more important goal, and detracts from the less important goal, it should pro tanto be adopted.
- If a solution for a case detracts from the more important goal, and promotes the less important goal, it should pro tanto be rejected.
- If a solution for a case promotes a goal to a large extent, this is pro tanto a stronger reason to adopt this solution than if it only minimally promotes the goal.
- If a solution for a case detracts from a goal to a large extent, this is pro tanto a stronger reason to reject this solution than if it only minimally detracts from the goal.

These guidelines demonstrate how the comparison of solutions for case types, given a set of goals, can be constructed in the form of QCR.

6. Comparing goal sets

It is not only possible to compare competing solutions for a type of cases in the light of a given set of goals, it is also possible to compare competing sets of goals in the light of a given normative system, that is in the light of a set of actual case solutions. To show how this can be done, I must briefly return to the justification of case solutions on the basis of a set of goals.

Given a set of goals, the solution for a particular case will promote some (zero or more) of these goals, detract from some other goals, and will be neutral with regard to the rest. Every goal which the solution for this case promotes, provides a reason for (the rightness) of the solution for this case, while every goal from which the solution detracts provides a reason against this solution. Whether the solution for a case is right all things considered, depends on the balance of these reasons. If the reasons why the solution is right (the pro-reasons) outweigh the reasons why the solution is wrong (the con-reasons), the solution is *right*. If the balance of reasons goes the other direction, the solution is *wrong*. If the reasons pro and con a conclusion are more or less in balance, the solution of the case is *indifferent.*

To make a decision about the rightness of the solution for a case, we must balance reasons and most often this will just be a matter of decision making. Such decisions about the relative weight of (sets of) reasons are expressed by what I will call *weighing knowledge*. Such weighing knowledge becomes also part of a theory of the law, and I will include it in the goal part of the theory.

Improvements in the goal part of a theory can take three forms, then. One is by making modifications in the set of goals by adding new goals or removing existing ones. The second is by making changes in the relative importance of the goals, and the third consists of changes in the weighing knowledge. The issue to be dealt with is when one of these changes is an improvement in the goal part of the theory.

Given a set of case solutions, it is possible to compare competing sets of goals (including relative importance and weighing knowledge) qualitatively. Every set of goals qualifies some of the actual case solutions as right, others as indifferent, and the rest as wrong. A set of goals A represents the actual case solutions better than another set B, if at least one of the actual case solutions is better in the light of A than it is in the light of B, while no actual solution is worse in the light of A than it is in the light of B. In other words, a change in the goal part of a theory is an improvement if at least one of the actual case solutions has changed from wrong into indifferent or right, or from indifferent into right, while no actual case decision has moved down one or more categories. If a solution that turns out to be wrong is seen as a reason against the goal set in the light of which this solution is wrong, and if a right solution is seen as a reason for the goal set, this way of comparing goal sets qualitatively is an application of the general technique of QCR described above.

7. Related work on theory construction

Legal theory construction, although not always under that name, is the topic of an enormous amount of literature. In a sense, most work on legal reasoning can be interpreted as dealing with legal theory construction. Concerning the logic of goals {legal principles, human rights), the work of Alexy (1979, 1996, 2000 and 2003) is particularly relevant. From the perspective of AI and law, work of McCarty

(1995), Berman and Hafner (1993), Sartor (2002), Bench-Capon and Sartor (2003), Prakken (2000 and 2002) and of myself (Hage 2000 and 2001) is most directly relevant.

In his paper on the implementation of Eisner versus Macomber, McCarty (1995) considered the situation where there are competing theories about the law, and raised the question how to determine which one is the best. In this connection he formulated the hypothesis that the answer '… depends on a theory of *coherence*. The task for a lawyer or a judge in a "hard case" is to construct a theory of the disputed legal rules that produced the desired legal result, and then to persuade the relevant audience that this theory is preferable to any theories offered by the opponent.' McCarty did not offer a theory of coherence in his paper, but the present work gives an indication what such a theory might look like.

In their (1993), Berman and Hafner made a point that is familiar to all students of legal reasoning, namely that the purpose of rules plays an important role in legal reasoning. From this observation they drew the conclusion that computational models of legal reasoning should take this purposive aspect of legal reasoning into account. From 2000 on, this theme received a lot of attention in the AI and Law literature (Bench-Capon 2000, Prakken 2000 and 2002, Bench-Capon and Sartor 2000 and 2003, Hage 2000 and 2001).

About my own earlier work (Hage 2000 and 2001) I can be brief. It is essentially what is presented here about theory construction, less adequately formalized because I tried to work with the old version of Reason-based Logic. The only (other) difference is that I paid attention to the relative importance of goals, assuming that contribution to a more important goal would lead to a weightier reason. From the point of view of QCR this is nothing new and for that reason it is not repeated here, although it illustrates another aspect of the fertility of QCR.

From a logical point of view, Prakken's papers (2000 and 2002) do not introduce anything new. The important contribution of these papers is that they illustrate how the formalism developed in, amongst others, Prakken and Sartor 1998, can be used to represent reasoning with values. The main idea is that an ordering of conflicting rules can be based on an ordering of values which these rules promote. This is essentially the same point as that made above, about the relation between the relative importance of goals and the relative weight of the reasons based upon them.

From 2000 to 2003, Bench-Capon and Sartor have, in a series of papers, independently and in co-operation, developed an impressive logical framework that deals with the combination of case-based reasoning, reasoning with values, theory construction and theory comparison. Like Prakken and the present author, they emphasized the role of values and their relative importance for the interpretation of (rules and) cases. Like Prakken and the present author, they used a mapping between the relative importance of values and the rules (reasons) related to them. Unlike Prakken and the present author, they include in their theory a substantial part about theory construction in the form of heuristics how to generate possible new theories of the law from old theories. Moreover, they pay some attention to the evaluation of theories, and they accept coherence as the relevant standard.

Relevant other differences with the present account of theory construction are that Bench-Capon and Sartor do not pay attention to the possibility that a rule frustrates a goal (as opposite of promoting it). Neither do they pay much attention to the fact that rules often promote (or frustrate) more than one goal (although they mention this point), or to the phenomenon that promotion and frustration of goals comes in degrees. These phenomena require that the so-called *accrual of reasons* is modelled.

Accrual of reasons touches on what is possibly the most important difference between the models of Prakken and Bench-Capon and Sartor on the one hand, and the model presented here on the other hand. Prakken, Bench-Capon and Sartor take their starting point in the logical system for defeasible reasoning developed by Prakken and Sartor, which is a system for argument comparison. A conclusion follows from a theory if it is the outcome of an argument that turns out to be justified in a battle of arguments. Conclusions are consequently tied to one particular argument, although it is not excluded that several justified arguments lead to the same conclusion. The combination, on a higher aggregation level, of arguments pleading for and against the same conclusion does not well fit in this model, however.[13]

When rules are compared with an eye toward several values, some of which they promote and others of which they frustrate, the combination of all these values is somehow important. A logical

[13] It may turn out to be possible to adapt the Prakken - Sartor logic to enable it to deal with the accrual of arguments, but I expect it to become similar to RBL then.

theory in which two rules are compared on the basis of two values which they respectively promote, can only handle the boundary case in which every rule in the comparison only promotes one goal and where the degrees of promotion are equal in all cases. My theory, which is based on RBL, takes the accrual of reasons as its natural starting point and is therefore better suited to deal with comparisons in which rules promote and frustrate several values at once, as is often the case.

8. Case-based reasoning as a form of comparative reasoning

Cases can be used in several ways in legal reasoning. One way, prevalent in the civil law tradition, is to extract a kind of rule from a decided case, and use this rule like other rules stemming from other legal sources. Another way is to use the case as a point of reference for an argument by analogy, or an e contrario argument. By pointing out an analogy between the old case and a new case, it is possible to argue that the decision taken in the old case should also be taken in the new case. Or, by pointing out a crucial difference between the old case and a seemingly similar new case, it is possible to distinguish the cases and to argue that there is no reason to copy the old decision in the new case, or even a reason to take a different decision.

The argument in which an analogy is drawn between two cases, in order to argue that the decision of the one case should be copied in the other case, can well be interpreted as a form of comparative reasoning. The way to do this is to compare cases with respect to their suitability for being decided in a particular way. If the old case was a suitable case for the decision that was actually taken in it, and the new case is just as suitable or even more suitable for such a decision, there is a reason to take this decision in the new case too. If the new case is less suitable than, or not well comparable to, the old case, this reason to decide the new case like the old one is lacking.

An example can illustrate this point. The following case was decided by the Dutch Supreme Court[14]:

> **Caustic soda case**
>
> Employees of a community centre placed a bag with household refuse along the street, in order to be taken away by the cleansing department. Unknown to the employees, the bag held a container with caustic soda. A cleaner put the bag into the dustcart, and due to some malfunctioning of the cart's mechanism, part of the caustic soda was swept into his face, as a consequence of which he suffered serious damages to his eyes. The cleaner sued the operator of the community centre for the damages.
>
> Even though the employees of the community centre were unaware of the presence of caustic soda in the bag, their behaviour was held to be negligent, because the court assumed a duty of care not to place a container with an unknown liquid in it, only protected by a cardboard box and a plastic bag, along the street to be taken away by the cleansing department, unless one has good reasons to assume that the liquid is not dangerous, or keeps the bag under control and warns those who want to handle the bag for its possibly dangerous contents.

Somewhat later the following case was brought before the Supreme Court[15]:

> **Yew case**
>
> Defendant's garden bordered on plaintiffs pasture, on which plaintiff held two horses. The pasture was fenced off by means of netting. Defendant had a heap of waste in his garden, near to plaintiffs meadow, on which he deposited a yew tree. Plaintiff's horses ate from the yew and died as a consequence. (Yew is poisonous for horses.) Plaintiff sues defendant for the damages. Defendant argued that he neither knew nor should have known that the yew was poisonous for horses.

In both cases the defendant created a dangerous situation to which plaintiff fell victim. Moreover, in both cases the risk for defendant was quite high, while the costs defendant had to make to avoid the danger were low. And, finally, the cases have in common that defendant was not aware of the danger he created. Given these similarities, it is well arguable that the cases should have similar decisions, and that therefore defendant in the second case should be held negligent as well.

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[14] HR 8-1-1982, NJ 1982, 614.
[15] HR 22-4-1994, NJ 1994, 624.

This is not what actually happened, however. The Dutch Supreme Court held that in the yew-case, defendant was not expected to know that yew is poisonous for horses. Under these circumstances, defendant was not held negligent. Apparently there is a legally relevant difference between on the one hand card boxes with an unknown content and on the other hand yew. In connection with card boxes with an unknown content, one should assume that the content may be dangerous, unless there are positive indications to the contrary, while in connection with yew, one does not have to take possible risks into account. This difference may be summarized by saying that in the caustic soda case the creation of the danger was recognisable, while in the yew-case the creation of danger was not recognisable. By pointing out this difference, the cases can be distinguished, with the result that in the one case defendant was held negligent, and in the other case he was not held negligent.

Let us look at both lines of argument in terms of comparative reasoning. I will start on the assumption that both cases are similar. In the following table, the columns labelled with a plus-sign contain the reasons that plead for negligence, while the columns labelled with a minus-sign contain the reasons against negligence.

Caustic Soda case			**Yew case**	
decision: defendant was negligent			**decision: ??**	
+	-		+	-
defendant created a dangerous situation to which plaintiff fell victim			defendant created a dangerous situation to which plaintiff fell victim	
it was easy and cheap to avoid the danger			it was easy and cheap to avoid the danger	
the potential damages were high			the potential damages were high	
	defendant was not aware that he created a danger			defendant was not aware that he created a danger

Since both cases have similar reasons pleading for and against the decision that defendant was negligent, they are prima facie equally suitable to support this decision. This would be different if the reasons in both cases had different weights. In the absence of evidence why this should be the case, one can work with the default assumption that similar reasons in different cases have equal weights. On this assumption, the cases are *equally suitable* to support the decision that defendant was negligent, and in combination with the fact that in the caustic soda case defendant was actually held negligent this is a reason why defendant should be held negligent in the yew case too.

Suppose, presumably counterfactually, that in the yew case the potential damages were even higher than in the caustic soda case. Then the reason based on the amount of damages in the yew case has a bigger weight than the similar reason in the caustic soda case. On the assumption that all other similar reasons have the same weights in both cases, the Yew case is then even *more suitable* for assuming negligence than the caustic soda case. *A fortiori* it then holds that there is a reason to assume negligence in the yew case, given that there was such a reason in the caustic soda case.

Let us now have a look at the cases from the point of view of the Dutch Supreme Court who found that in the caustic soda case, defendant should have taken the possible danger into account, while in the yew case this was not the case.

Caustic Soda case			**Yew case**	
decision: defendant was negligent			**decision: ??**	
+	-		+	-
defendant created a dangerous situation to which plaintiff fell victim			defendant created a dangerous situation to which plaintiff fell victim	
it was easy and cheap to avoid the danger			it was easy and cheap to avoid the danger	
the potential damages were high			the potential damages were high	
	defendant was not aware that he created a danger			defendant was not aware that he created a danger
defendant should have been aware that he created a danger				

On this reading of the cases, the caustic soda case had one reason to assume negligence that was lacking in the yew case. In all other respects the cases were similar. On this reading, the caustic soda case is more suitable for the assumption of negligence than the yew case. As a consequence the reason to decide the cases similarly that was present in the other reading of the cases is lacking on the present reading.

Notice that this reason (based on similarity of the cases) would still be absent if the reasons pro negligence in the yew case would weigh more than the similar reasons in the caustic soda case, for instance because the potential damages were higher in the yew case, because then both cases would in one respect be more suitable for the assumption of negligence, and that makes comparison by means of qualitative reasoning impossible.[16]

As the above example illustrates, at least some form of case-based reasoning can be interpreted as a special case of QCR, namely as the comparison of cases with respect to their suitability for a particular solution. Obviously, theory construction is also relevant in connection with cases. When case-based reasoning is used as a technique, the solution is kept fixed, and cases are compared with regard to their suitability for *this solution.* When theory construction is used as a technique, a case is kept fixed as a point of reference, while solutions are compared with regard to their suitability for *this case.*

These two techniques can also be combined. Given a particular case, it is possible to compare possible solutions with regard to their suitability. When a particular solution has been adopted as, given the available information, the best one, it is possible to compare actual and hypothetical cases

[16] At least in the absence of additional relevant information.

with regard to their suitability for this solution. In this way the best solution for one type of case can be transferred to other cases, thereby broadening the theory of the law that is under construction.

9. Related work on case comparison

The work on case-based reasoning in AI and law can be divided into work that deals with case comparison, which is relevant for QCR, and the rest, which is not relevant from the perspective of QCR. For instance, the interesting paper by Prakken and Sartor about case-based reasoning (Prakken and Sartor 1998) does not deal with case comparison, but rather with the justification of rules on the basis of cases

The core texts with regard to case comparison are still Ashley 1990, 1991 and 1992. Ashley's line of research was continued in Aleven 1997. All of this work is mainly focussed on the creation of a computer program in which case comparison plays a central role. The logical theory underlying the programs remained relatively underdeveloped. An early attempt to make this underlying logic explicit can be found in Hage 1997, section V,9. Roth 2003 provides the as yet most exhaustive account of the logic of case comparison.

The use of QCR for case-based reasoning, as described in this paper is more of a re-interpretation of the work of Ashley and Roth than a radical new approach. The contribution of this paper to the theory of case comparison is that it shows how the existing theory of case comparison can be seen as a special case of QCR, with the additional effect that it suggests how the general insights about QCR can be used to develop the theory of case comparison even further. The brief remarks at the end of the previous section about the relation between case-based reasoning and theory construction suggest how the re-interpretation of the logic of case comparison can generate interesting research questions.

10. Qualitative comparative reasoning and legal proof

Legal proof is the subject of a tremendous amount of legal literature, e.g. Twining 1985, 1991 and Tillers and Green 1988. Recently, the attention for this field in AI and law in increasing (e.g. Prakken e.a. 2003, Bex e.a. 2003, and Prakken 2004; see also Verheij 2003). This work is basically the application of existing theories of defeasible legal reasoning to reasoning about evidence. To my knowledge (supported by the expert advice of H. Crombag), comparative reasoning as a tool for reasoning about proof has not been the subject of much discussion yet, neither in the law, nor in AI and law. In the rest of this section, I will try to show that QCR can also play a role in connection with legal proof.

When there are several competing accounts of the facts about a case, these accounts can be compared with regard to how well the fit the evidence. I cannot go deep into this issue here, but let me illustrate it by means of an example. Suppose that Lord Hard was found in his room, murdered by means of a knife. The butler was seen by John, the Lord's son, when the butler allegedly entered Lord Hard's room at about the estimated time of the killing. Moreover, the butler had a motive to murder Lord Hard, because his Lordship had seduced the butler's daughter Harriet. However, there is also a witness, the gardener, who testifies that the butler was in the garden at the estimated time of the murder.

There is also another suspect, the chamber maid Dorothea, who also had a motive to murder Lord Hard, because she had a relationship with the Lord before he fell in love with Harriet, and she suffered severely from jealousy. Dorothea was also seen by John when she allegedly entered Lord Hard's room at about the estimated time of the killing. The problem is, however, that Dorothea has an alibi too in the form of a visiting grocer who delivered some goods to Dorothea at the time in question. Schematized, the two competing theories have the following reasons pleading for and against them:

The butler committed the murder			**The maid committed the murder**	
+	-		+	-
motive			motive	
witness that the butler had the opportunity			witness that the maid had the opportunity	
	alibi			alibi

At first sight the two theories are equally good. However, if the information is added that the gardener is the butler's brother, the value of the butler's alibi becomes considerably less. If it may be assumed that if the gardener is the butler's brother he may have lied about the presence of the butler in the garden at the estimated time of the murder. In terms of reasons, it may be said that the butler's alibi as a reason against the theory that he committed the murder has less weight than the alibi of Dorothea, which was based on a more reliable witness. (I represented this in the above schema by giving the butler's alibi a smaller font.) Assuming that the motives of the butler and of Dorothea were equally strong (had the same weight) and that the testimony of John was equally reliable with regard to the butler and to the maid, the theories that the butler killed the Lord and that the maid killed him are equally strong in pro-reasons. However, the theory that the butler committed the murder is weaker in con-reasons than its competitor and should therefore be preferred (in the absence of additional information).

Obviously, a similar result can be achieved by removing one of the reasons why the maid committed the murder. If John did not see the maid enter Lord Hard's room, the theory that the maid killed Lord Hard is worse than the theory that the butler killed him, even if we disregard the fact that the butler's alibi was provided by his brother.

In general, competing theories about what happened in a concrete case can be compared in terms of sets of reasons pleading for and against them. These reasons will (at least partly) be based on pieces of evidence that must be explained by the theory.

A piece of evidence that is explained provides a pro-reason for the theory. The better the explanation is, the stronger is the pro-reason. A piece of evidence is explained marginally if its existence is compatible with the theory. It is explained better if its existence is a plausible consequence of the theory, and it is explained still better if the truth of the theory necessitates its presence. For instance, the theory that the butler killed Lord Hard explains the testimony of John, because this theory explains why the butler entered the Lord's room, and thereby that John could see the butler entering the room, and thereby that John testified that he saw the Lord entering this room. The explanation would even be better if the theory entailed that John had to (instead of merely could) see the butler entering the room.

A pro-reason also becomes stronger if the evidence it explains is more reliable. If, for instance, John held a grudge against the butler, the pro-reason for the theory that the butler was the murderer would be weaker than it actually is.[17]

A piece of evidence that the theory fails to explain and that requires explanation[18] provides a con-reason for a theory. The more remarkable the lack of explanation is, the stronger is the con-reason. What was written in connection with pro-reasons about the strength of the reasons holds *mutatis mutandis* also for con-reasons.

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[17] This can also be accounted for by the observation that there is another explanation for John's testimony which makes the explanation by the theory that the butler killed the Lord less plausible.

[18] Not all facts of a case need to be explained by a theory about a case that is necessarily incomplete. However, some facts of a case are remarkable and seem to require a special explanation. It is these facts that lead to counterevidence for theories that do not explain them.

When the information about the available evidence is translated along the above-mentioned lines into sets of reasons for and against competing accounts of what happened, the general theory bout QCR can be used to compare these accounts and pick out the best one (if there is a best one).

B. REASON-BASED LOGIC

Qualitative comparative reasoning as described above deals with the comparison of sets of reasons. It is therefore possible and in the present author's eyes attractive to use Reason-based Logic (RBL) the logic that was specially developed for reasoning with reasons to formalize the above account QCR. However, the versions of RBL as described in Hage 1996 and Hage 1997 are not suitable, because for QCR it is necessary to deal with *similar reasons*, a notion that cannot easily be incorporated into these older versions of RBL. As a consequence it turned out to be necessary to reconceptualize RBL by assigning an important role to the notion of an abstract reason, and by removing the notion of a principle from the theory. The resulting new version of RBL, which is much more powerful because it can deal not only with reasons for and against one particular conclusion but also with the comparison of a number of alternatives, is described below. To make the paper understandable for an audience not familiar with the previous work on RBL, I have decided to repeat some material that was published in Hage 1996 and 1997.

11. The language and ontology of RBL

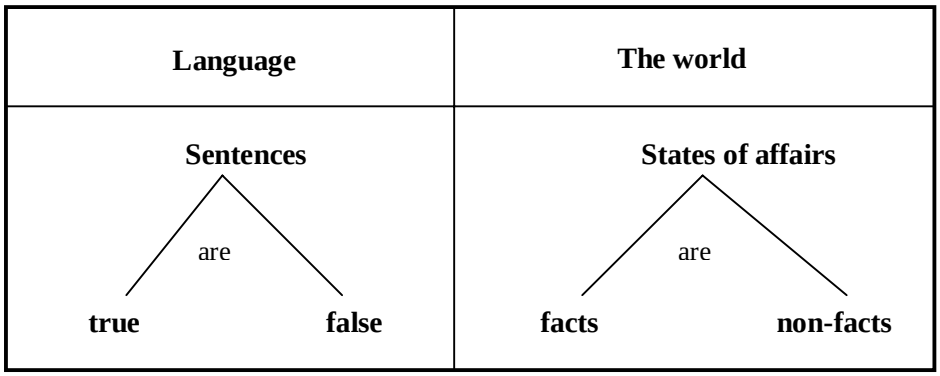
One important way in which RBL is an extension of predicate logic is that its language is an extension of the language of predicate logic. The extension consists mainly in a number of dedicated predicates, relations, and function expressions that play a logical role in RBL. In this and the following sections, these extensions are introduced in an explanatory context. The first conventions concern a specification of the language for predicate logic that will be used:

- All constants for relations, predicates, and sentences without a subject-predicate structure start with an uppercase letter.
- All function expressions, individual constants and variables start with a lowercase letter, except individual constants and variables that denote states of affairs, which start with an asterisk (*), followed by a lowercase letter.
- The constants $\forall$, $\exists$, $\sim$, &, $\vee$, $\rightarrow$ and $\equiv$ stand for the universal and the existential quantifier, negation, conjunction, inclusive disjunction, the material conditional and equivalence, respectively.
- Variables are *italicized*.

11.1. Sentences, states of affairs and facts

RBL presupposes a rich ontology. Next to the 'ordinary' physical things, it also assumes several kinds of immaterial entities, including states of affairs and sets of individuals (in particular sets of reasons). States of affairs are what is expressed by sentences with truth values.[19] For instance, the sentence 'It's raining' expresses the state of affairs that it is raining. Some states of affairs *obtain* in the world; these are called *facts*. A sentence that expresses a fact is *true*. False sentences express *non-facts*, that is states of affairs that do not obtain.

[19] The clause 'with truth values' is meant to exclude non-descriptive sentences, such as commands, but also descriptive sentences that have terms on referential positions that have no object of reference, such as 'The king of France is bald'. Cf. Strawson 1950.



In most declarative sentences it is possible to distinguish one or more terms whose function it is to refer to entities in the world. Next to these terms there will be a predicate expression by means of which something is said about the entities referred to. For instance, in the sentence 'John walks' the word 'John' refers to John, and the word 'walks' is used to say something about John. In the sentence 'Jane gave the book to the father of Mary' the expressions 'Jane', 'the book', 'Mary', and 'the father of Mary' refer, while 'gave … to' indicates the relation between the three entities referred to. The expression 'the father of Mary' is a so-called function expression. It refers itself (to the father of Mary), but it also contains the term 'Mary', which refers to Mary.

Logicians call the entities about which a sentence is *(logical) individuals*, and the expressions used to refer to them *terms*. Terms should be distinguished from full sentences. Sentences have truth values, terms not. Even function expressions, although they contain a reference to an individual, have no truth values. The reason for this is clear: function expression refer to individuals; they do not state anything. So, there is, from a logical point of view, a fundamental difference between sentences and terms. Sentences have truth values; they do not refer. Terms, on the contrary, have no truth values, but refer.

By assuming the existence of states of affairs, this clear distinction between on the one hand terms and individuals, and on the other hand sentences and truth values, is blurred somewhat. Sentences no longer only have truth values, they can also be treated as terms that denote the states of affairs expressed by them. This happens, for instance, in sentences that deal with so-called propositional attitudes, such as 'Mary beliefs that John walks'. Taken by itself, the sentence 'John walks' has, in its quality of a sentence, a truth value, but as content of a propositional attitude it refers to the state of affairs that John walks.

In RBL the distinction between the two functions of sentences is made explicit by syntactical means. The state of affairs expressed by sentence `S` is *typically* denoted by the term `*s`. In this way, a term that typically denotes a state of affairs indicates by its internal structure which state of affairs it denotes. Since states of affairs are logical individuals, they can also, non-typically, be denoted by other terms. For instance, the state of affairs `*it's_raining` can also be denoted by the term `*a`. In that case the sentence `*a = *it's_raining` is true.[20] In general the following translation holds between sentences and the terms that typically denote the states of affairs expressed by these sentences:

- If `S` is a sentence, and if `s` is the string that results if all the uppercase letters at the beginnings of the atomic sentences that are part of `S` are replaced by lowercase sentences, then `*s` typically denotes the state of affairs expressed by `S`.
- If `*s` is a term typically denoting a state of affairs, and `S` is the sentence that results if all the lowercase letters at the beginnings of terms denoting atomic states of affairs are replaced by uppercase letters, then `S` expresses the state of affairs denoted by `*s`.

Variables for states of affairs start with an asterisk too. For instance, the following sentence expresses that Jane believes everything that John believes:

$\forall$`*s(Believes(john, *s)` $\rightarrow$ `Believes(jane, *s))`

[20] The convention that terms denoting states of affairs start with an asterisk is also used for terms and variables that non-typically denote states of affairs.

If a sentence is true, the state of affairs expressed by it obtains. RBL has in this connection a dedicated predicate constant `Obtains/1`, that operates on terms that denote states of affairs. The relation between the truth of a sentence and the state of affairs expressed by this sentence is rendered by the following axiom of RBL:

Definition obtains:
$\forall$*s(Obtains(*s)) ≡ S

11.2. Abstract states of affairs

States of affairs are either abstract or concrete. An abstract state of affairs can be realized (instantiated) in different ways. For instance, the abstract state of affairs that it is raining can be realized at different times and places. The abstract states of affairs that somebody gives something to somebody else is realized by the concrete state of affairs that John gives Mary a book.

Abstract states of affairs are denoted by a term for a state of affairs that contains at least one free variable. Examples are:

- *rescued(tarzan, *y*) denotes the abstract state of affairs that Tarzan rescued somebody. Notice that this expression is a term that denotes a state of affairs. In particular it should be distinguished from the sentence ($\exists$*y*)Rescued(tarzan, *y*), which expresses the concrete state of affairs that there is a person that Tarzan rescued and from the term *($\exists$*y*)Rescued(tarzan, *y*)which denotes this last concrete state of affairs.
- *gives(*x*, a_book, *y*) & (*x* ≠ *y*) denotes the abstract states of affairs that somebody gives a book to somebody else.

Concrete states of affairs can instantiate abstract ones. A concrete state of affairs *s instantiates an abstract state of affairs *s', if and only if there is some substitution i such that the term that typically denotes *s is the result of uniformly substituting all variables in the term that typically denotes *s' by constants according to i.

Whereas states of affairs can be both abstract and concrete, facts, states of affairs that actually obtain, are always concrete.

12. Reasons

The central notion in RBL is that of a reason. There are several different kinds of reasons, which have in common that they are facts which are relevant for other facts. These other facts are called the *conclusions* of these reasons.

12.1. Decisive reasons

RBL distinguishes between *contributive reasons* and *decisive reasons*. Decisive reasons are concrete reasons[21] that determine their conclusions. If a decisive reason for a conclusion obtains, the conclusion also obtains. For instance the facts that there are two horses and four cows and no other animals, are together a decisive reason why there are six animals. It makes no sense to weigh decisive reasons against other reasons. If there are contributing reasons that collide with a decisive reason, the decisive reason wins by definition, so there is no need for weighing. If there would be colliding decisive reasons, this would make their conclusions incompatible. Since there cannot be incompatible states of affairs, there cannot be colliding decisive reasons either.

RBL has a dedicated predicate constant `Dr/2` to express that a fact is a decisive reason for some other fact. For instance, the following sentence expresses that the fact that John is older than John is a decisive reason why John is younger than John:

```
Dr(*older_than(john, john), *younger_than(john, john))
```

Because all reasons are facts, it holds that:

Decisive reason is fact:
$\forall$*a($\exists$*b(Dr(*a,*b)) → Obtains(*a))

[21] The distinction between concrete and abstract reasons is discussed in section 11.5.

Moreover, because a decisive reason guarantees that its conclusion obtains, it holds that:

Conclusion decisive reason obtains:
$\forall *b(\exists *a(\mathrm{Dr}(*a, *b)) \rightarrow \mathrm{Obtains}(*b))$

12.2. Contributive reasons

Just like decisive reasons, contributive reasons are concrete reasons. In opposition to decisive reasons, contributing reasons do not determine their conclusions by themselves. There can both be contributing reasons that plead for, and contributive reasons that plead against a particular conclusion. Assuming that there are no relevant decisive reasons, it is the set of *all* contributing reasons concerning a particular conclusion, both the reasons pro and con, that determines whether the conclusion holds.

For instance, if somebody breaks the window of somebody else's house in order to save a child from the house that is burning, the question whether this behaviour is lawful depends on the relative weight of two contributing reasons. One reason is that the behaviour was an infringement of somebody else's property. This reason pleads *against* the lawfulness of the behaviour. The other reason is that the act was necessary to save a human life. This reason pleads *for* the lawfulness of the behaviour. If these two are all the relevant reasons, they determine together whether the behaviour in question was lawful.

RBL has a dedicated predicate constant `Cr/2` to express that some fact is a contributive reason for some state of affairs. For instance the following sentence expresses that the fact that John is a thief is a contributive reason why he is punishable:

```
Cr(*thief(john), *punishable(john))
```

There can also be contributive reasons against a conclusion. To avoid the necessity of a special predicate constant for the expression of contributive reasons against a conclusion, these *con-reasons* are expressed as if they were *pro-reasons* for the negated conclusion. For instance, the fact that the weather report predicted sunshine is a contributive reason against the conclusion that it will be raining:

```
Cr(*prediction_sunshine, *~rain)
```

Because all contributive reasons are facts, it holds that:

Contributive reason is fact:
$\forall *a(\exists *b(\mathrm{Cr}(*a, *b)) \rightarrow \mathrm{Obtains}(*a))$

12.3. Weighing contributive reasons

A crucial aspect of contributive reasons is that *they have to be weighed* (or *balanced*; I use these words interchangeably) against contributive reasons pleading in a different direction. To avoid misunderstandings, I want to stress that this weighing needs not to be a psychological process. From a logical point of view it does not matter what goes on psychologically. The only thing that matters that somehow information must become available that indicates which set of reasons outweighs the other set. This information needs not be available in advance to guide a decision making process; it can also be the outcome of such a process. From the logical point of view, information about the relative weights of the sets of reasons is merely a presupposition of a valid argument in which a conclusion is drawn from contributive reasons for and/or against this conclusion.

To formalize this kind of argument, RBL needs function constants to denote sets of reasons. The function constant $\mathrm{r}^{+}/1$ denotes the set of contributive reasons pleading for a conclusion, while $\mathrm{r}^{-}/1$ denotes the set of contributive reasons pleading against a conclusion.

If `*a` denotes a concrete state of affairs, then $\mathrm{r}^{+}(*a)$ denotes the set of all contributive reasons pleading for *a:

Definition set of contributive pro-reasons:
$\mathrm{r}^{+}(*a) = \{*s \mid \mathrm{Cr}(*s, *a)$ is true$\}$.

$\mathrm{r}^{-}(*a)$ denotes the set of all contributive reasons pleading against *a:

Definition set of contributive con-reasons:
$\mathrm{r}^{-}(*a) = \{*s \mid \mathrm{Cr}(*s, *\sim a)$ is true$\}$.

For example, $r^+(*rain)$ denotes the set of all reasons why there will be rain, while $r^-(*rain)$ denotes the set of all reasons why there will not be rain.

As mentioned, contributive reasons need to be weighed. Psychologically, this weighing often boils down to taking a decision which set of reasons outweighs the other set. However, as already stressed, from the logical point of view the only thing that matters is that information about the relative weight is needed as a premise in a valid argument.

The information which set of reasons outweighs the other set is expressed in so-called *weighing knowledge*. RBL has a dedicated relation constant to express weighing knowledge: $>_{conclusion}/2$, which operates on sets of reasons. For instance, the sentence

$\{*prediction\_sunshine >_{*rain} \{*cloudy, *rain\_yesterday\}$

expresses that the set containing the single reason that sunshine was predicted by the weather forecast, with regard to the conclusion that it will rain, outweighs the set containing the reasons that it is cloudy and that it rained yesterday.

Often the subscript of the Outweighs-predicate is superfluous because the context makes it clear for and against which conclusion the reasons plead. Then the subscript is omitted.

If the contributive reasons pleading for a conclusion outweigh the contributive reasons pleading against it, and there is no decisive reason against this conclusion, the conclusion holds:

Outweighing pro-reasons:

$\forall *s(r^+(*s) > r^-(*s) \ \& \ \sim\exists *x(Dr(*x, *\sim s)) \rightarrow Obtains(*s))$

From this it follows that if the contributive reasons pleading against a conclusion outweigh the contributive reasons pleading for it, and there is no decisive reason for this conclusion, the negation of the conclusion holds:

Outweighing con-reasons:

$\forall *s(r^-(*s) > r^+(*s) \ \& \ \sim\exists *x(Dr(*x, *s)) \rightarrow Obtains(*\sim s))$

12.4. Abstract reasons

If a particular fact is a reason for some conclusion, similar facts will normally be reasons for similar conclusions. Suppose, for instance, that the fact that the weather forecast this morning predicted that it will rain this afternoon is a reason to assume (why it is rational to assume) that it will rain this afternoon. In that case the fact that in the morning the weather forecast predicts rain will in general be a reason to assume that it will rain that afternoon. Or, being a thief is an abstract contributive reason for being punishable, because the fact that some particular person is a thief is normally a contributive reason why this person is punishable.[22]

In RBL, all contributive reasons are considered to be instantiations of abstract reasons. RBL has a dedicated predicate constant $Ar/2$ to express that some abstract state of affairs is an abstract contributive reason for some (other abstract) state of affairs. For instance the following sentence expresses that being a thief is a contributive reason for being punishable:

$Ar(*thief(x), *punishable(x))$

If some fact instantiates an abstract contributive reason for some abstract conclusion, this fact is normally a concrete contributive reason for the relevant instantiation of the abstract conclusion. For instance, if

$P(a) \ \& \ Ar(*p(x), *c(x))$

is true, then

[22] In my 1996 and 1997 I hardly paid attention to abstract reasons and instead wrote about (the validity of) principles. This may have created the wrong impression that these principles would possess some kind of independent status apart from that some type of fact tends to be a reason for some type of conclusion. (E.g. as so-called legal principles'.) To avoid this impression, I discuss this topic here in terms of abstract and concrete reasons.

`Cr(*p(a), *c(a))`

will *normally* also be true.

Sometimes instantiations of abstract reasons are not contributive reasons. This happens in particular if there is a reason why a kind of fact that is normally a reason for some conclusion should not be taken into consideration. Take the following example: John has promised his mother in law to visit her on Sunday afternoon. After the promise, he finds out that there will be a unique concert by his favourite artist that same afternoon. Normally, John would have to balance the obligation stemming from the promise against his desire to visit the concert in order to decide what he should do. John happens to find out, however, that his mother in law will have other visitors too that afternoon, while she gave him the impression that she would be all alone if John would not visit her. Knowing this, John does not feel bound by his promise anymore, and there is no need to balance reasons in order to decide what to do. The only relevant reason is the unique concert, and because of this reason John decides to visit the concert.

When the step from an abstract reason to its instantiation as contributive reason is not valid, we say that the abstract reason is *excluded*. The language of RBL uses the predicate constant `Excluded/2` to express such an exclusion. Its parameters are the relevant instantiations of the abstract reason and its conclusion. For instance, if `Ar(*p(`x`), *c(`x`))` is true, `Excluded(*p(a), *c(a))` expresses the lack of relevance of this abstract reason for the instantiation of x by `a`. If an abstract reason is not excluded, its instantiation is a contributive reason:

Instantiation of abstract reason:
Let `*ip` be an instantiation of `*p` under some substitution of the variables in `*p` and let `*ic` be the instantiation of `*c` under the same substitution of variables. Then it holds that:

`Ar(*p, *c) & Ip & ~Excluded(*ip, *ic)` $\rightarrow$ `Cr(*ip, *ic)`

The exclusion of abstract reasons illustrates a phenomenon that occurs more often, namely that states of affairs of a particular type only obtain if there are special reasons for it. Such abstract states of affairs may be called *reason-based states of affairs*.[23] In general an abstract state of affairs is reason-based if its instantiations can only obtain if either there is a decisive reason for it, or the contributive reasons pleading for it outweigh the contributive reasons pleading against it:

Definition reason-based states of affairs:
Let `*as` denote an abstract state of affairs and let `*s` be an instantiation of `*as`. Then it holds that
$\forall$`*as(Reason_based(*as)` $\equiv$ $\forall$`*`s`(`$\exists$`*`r`(Dr(*`r`, *`s`))` $\vee$ r^+`(*`s`)` > r^-`(*`s`))`

Exclusion reason-based:
Let `*r` denote a concrete reason for the state of affairs denoted by `*c`. Then it holds that
$\forall$`*`r`,*`c`(Reason-based(*excluded(*`r`,*`c`)))`
(The type of fact that an abstract reason is excluded is reason-based.)

12.5. Weighing knowledge

Weighing knowledge is in general contingent, but there are two exceptions concerning the empty set. An empty set of reasons is normally[24] outweighed by any non-empty set, and it does normally not outweigh any set. This is expressed by the following abstract reasons:

Empty set of pro-reasons outweighed by non-empty set of con-reasons:
$\forall$`*`c`(Ar(*`r^+`(*`c`)` = $\emptyset$ & r^-`(*`c`)` $\neq$ $\emptyset$, `*`r^-`(*`c`)` > r^+`(*`c`)))`

---

[23] Another example of a reason-based state of affairs is the state of affairs that an actor is obligated to perform some kind of behaviour. There are no obligations without reasons. Cf. Hare 1963, p. 30f.

[24] There are situations where single reasons do not allow the derivation of the conclusions for which they plead. For instance the fact that the suspect has a motive for committing the murder is a reason to believe that this suspect was the murderer. However, this reason is in itself not sufficient, even in the absence of counter reasons, to draw the conclusion that the suspect committed the murder. Situations like these are exceptions to the principle that any non-empty set of reasons outweighs the empty set.

Empty set of con-reasons outweighed by non-empty set of pro-reasons:

$\forall *c(\mathrm{Ar}(*r^{-}(*c) = \varnothing \;\&\; r^{+}(*c) \neq \varnothing, *r^{+}(*c) > r^{-}(*c)))$

Empty set of pro-reasons does not outweigh any set of con-reasons:

$\forall *c(\mathrm{Ar}(*r^{+}(*c) = \varnothing, *\sim(r^{+}(*c) > r^{-}(*c))))$

Empty set of con-reasons does not outweigh any set of pro-reasons:

$\forall *c(\mathrm{Ar}(*r^{-}(*c) = \varnothing *\sim(r^{-}(*c) > r^{+}(*c))))$

13. Extension of RBL for Qualitative Comparative Reasoning

The basic version of RBL exposed in the above section needs to be extended to make it suitable for dealing with QCR.

13.1. Similar states of affairs and reasons

Sets of reasons that plead for the 'same' conclusion in different cases do not really contain the same reasons, but only reasons that are 'similar' to each other. The following definitions deal with similarity.

Similar states of affairs

Two concrete states of affairs are said to be similar if and only if they are instantiations of the same abstract state of affairs:

$$\forall *s1,*s2(\mathrm{Similar}(*s1,*s2) \equiv \exists *s3,i1,i2(s1 = \mathrm{Instantiation}(s3, i1) \;\&\; s2 = \mathrm{Instantiation}(s3, i2)))$$

Notice that identical states of affairs are also similar states of affairs.

Similar reasons

Analogous to the definition of similar states of affairs, two concrete reasons are similar if and only if they are both instantiations of the same abstract state of affairs, and their conclusions are also similar states of affairs. Formally:

$$\forall *a1,*a2(\mathrm{Similar\_reasons}(*a1, *a2) \equiv \exists *c1,*c2(\mathrm{Cr}(*a1,*c1) \;\&\; \mathrm{Cr}(*a2,*c2) \;\&\; \mathrm{Similar}(*a1,*a2) \;\&\; \mathrm{Similar}(*c1,*c2)))$$

Similar reasonsets

Sets that consist of pair wise similar reasons are called similar reason sets:

$$\forall a,b(\mathrm{Similar\_reasonsets}(a, b) \equiv \forall *s1((*s1 \in a) \rightarrow \exists s2((*s2 \in b) \;\&\; \mathrm{Similar\_reasons}(s1, s2))) \;\&\; \forall *s2((*s2 \in b) \rightarrow \exists s1((*s1 \in a) \;\&\; \mathrm{Similar\_reasons}(s1, s2))))$$

Similar superset

Given the notion of similar reasons, it is also possible to define the notion of a 'similar superset'. A similar superset is like a proper superset, with the difference that all elements of the subset must have a *similar* reason in the superset. Formally:

$$\forall a,b(\mathrm{Similar\_superset}(a, b) \equiv \forall *s2((*s2 \in b) \rightarrow \exists *s1((*s1 \in a) \;\&\; \mathrm{Similar\_reasons}(*s1,*s2))) \;\&\; \exists *s3 ((*\mathrm{s3} \in a) \;\&\; \forall *s4((*s4 \in b) \rightarrow \sim\mathrm{Similar\_reasons}(*s4,*s3))))$$

Similar subset

A similar subset is like a proper subset, with the difference that all elements of the subset must have a *similar* reason in the superset. Formally:

```
∀a,b(Similar_subset(a, b) ≡
   ∀*s2((*s2 ∈ a) → ∃*s1((*s1 ∈ b) & Similar_reasons(*s1,*s2))) &
   ∃*s3 ((*s3 ∈ b) & ∀*s4((*s4 ∈ a) → ~Similar_reasons(*s4,*s3))))
```

By means of the notions of a similar reason set, a similar superset and a similar subset it is possible to overcome the problems connected with the fact that the sets that must be compared do not contain identical, but merely 'similar' reasons.

13.2. The weight of reasons

Another issue that must be dealt with is that of comparing sets of reasons on their weights. The idea to be captured is that if a set contains a similar reason for every reason in the other set, while from each pair of similar reasons, the reason in the former set does not have a smaller weight than its counterpart in the latter set, while at least one reason has a bigger weight than its counterpart, the former set is *stronger in individual weight* than the latter.

Weight

The first step to take in this connection is to define a function that maps reasons on their weights. Let `*r` be a contributive reason for conclusion `*c`. Then `weight(*r,*c)` denotes the weight of `*r` as a contributive reason for `*c`.

Two similar reasons have in principle the same weight. This can be expressed as follows:

```
Ar(*cr(*r1,*c1) & cr(*r2,*c2) & similar(*r1,*r2) &
   similar(*c1,*c2), *weight(*r1,*c1) = weight(*r2,*c2))
```

`>/2 and </2`

The second step is to assign a second meaning to the relations `>/2` and `</2`.[25] These relations hold between the weights of two reasons if and only if the weight of the first reason is bigger, respectively smaller than the weight of the second reason. For instance:

```
weight(*r1,*c1) > weight(*r2, *c2).
```

Comparable reason sets

The third step is to define the relation *stronger in individual weight*, which can hold between sets of reasons. These sets must either both contain reasons that plead for a similar conclusion, or reasons that plead against a similar conclusion. I will call such sets *comparable reason sets*.

```
∀s1,s2 (Comparable_reasonsets(s1,s2) ≡
   ∃*c(s1 ⊆ r+(*c) & s2 ⊆ r+(*c)) ∨
   ∃*c(s1 ⊆ r-(*c) & s2 ⊆ r-(*c)))
```

$>_w/2$

The relation *stronger in individual weight* ($>_w/2$)holds between two comparable reason sets, if and only if from the reasons which the two sets *have in common* at least one reason of the first set weighs more than the corresponding reason from the second set, while the opposite is not the case.

[25] The first meaning is that of the outweighs-relation that holds between sets of contributive reasons for and against the same conclusion.

Formally:

$\forall s1,s2((s1 >_w s2) \equiv$
Comparable_reasonsets(s1,s2) &
$\exists *r1,*r2,*c1,*c2($
Cr($*r1$, $*c1$) & ($*r1 \in s1$) &
Cr($*r2$, $*c2$) &($*r2 \in s2$) &
Similar_reasons($*r1$, $*r2$) &
(weight($*r1$,$*c1$) > weight($*r2$,$*c2$)) &
$\forall *r3,*r4,*c3,*c4($
Cr($*r3$, $*c3$) & ($*r3 \in s1$) &
Cr($*r4$, $*c4$) &($*r4 \in s2$) &
Similar_reasons($*r3$, $*r4$) $\rightarrow$ ~(weight($*r4$,$*c4$) > weight($*r3$,$*c3$)))

$<_w/2$

The relation *weaker in individual weight* ($<_w/2$) holds between two comparable reason sets, if and only if from the reasons which the two sets *have in common* at least one reason of the first set weighs less than the corresponding reason from the second set, while the opposite is not the case.

Formally:

$\forall s1,s2((s1 <_w s2) \equiv$
Comparable_reasonsets(s1,s2) &
$\exists *r1,*r2,*c1,*c2($
Cr($*r1$, $*c1$) & ($*r1 \in s1$) &
Cr($*r2$, $*c2$) &($*r2 \in s2$) &
Similar_reasons($*r1$, $*r2$) & (weight($*r1$,$*c1$) < weight($*r2$,$*c2$)) &
$\forall *r3,*r4,*c3,*c4$(Cr($*r3$, $*c3$) & ($*r3 \in s1$) &
Cr($*r4$, $*c4$) &($*r4 \in s2$) &
Similar_reasons($*r3$, $*r4$) $\rightarrow$
~(weight($*r4$,$*c4$) < weight($*r3$,$*c3$)))

$=_w/2$

The relation *equal in individual weight* ($=_w/2$) holds between two comparable reason sets, if and only if all the reasons which the two sets *have in common* pair wise have equal weights.

Formally:

$\forall s1,s2((s1 =_w s2) \equiv$
Comparable_reasonsets(s1,s2) &
$\forall *r3,*r4,*c3,*c4($
Cr($*r3$, $*c3$) & ($*r3 \in s1$) &
Cr($*r4$, $*c4$) &($*r4 \in s2$) &
Similar_reasons($*r3$, $*r4$) $\rightarrow$
(weight($*r4$,$*c4$) = weight($*r3$,$*c3$))))

Notice that it is not necessarily the case that between two comparable reason sets one of the relations stronger than, weaker than, or equal in individual weight holds.

Stronger

There are two requirements if a set of reasons is to be stronger than another set *on logical grounds*, based on the notion of a similar superset, and on that of being stronger in individual weight. These two must be combined to obtain the result that a set is stronger than another set. In this connection there are two possibilities:

1. the first set is a similar superset of the second, while the second is not stronger in individual weight;
2. the first set is stronger in individual weight than the second, while the second is either a similar reason set of a similar subset of the first.

Formally:

$$\forall s1,s2(\text{similar\_superset}(s1, s2) \,\&\, \sim(s2 >_w s1)) \vee ((s1 >_w s2) \,\&\, \text{similar\_reasonset}(s2, s1) \vee \text{similar\_subset}(s2, s1)) \rightarrow \text{Stronger}(s1, s2))$$

Weaker

There are two logical grounds for which a set of reasons can overall be weaker than another set, namely if the latter is a similar superset of the former, or if it is weaker in individual weight. These two must be combined to obtain the result that a set is weaker than another set. In this connection there are two possibilities:

1. the second set is a similar superset of the first, while the second is not weaker in individual weight;
2. the first set is weaker in individual weight than the second, while the first is not a similar superset of the second.

Formally:

$$\forall s1,s2(\text{similar\_superset}(s2, s1) \,\&\, \sim(s2 <_w s1)) \vee ((s2 >_w s1) \,\&\, \text{similar\_reasonset}(s1, s2) \vee \text{similar\_subset}(s1, s2)) \rightarrow \text{Weaker}(s1, s2))$$

Equal

A set of reasons is equal to another set on logical grounds if:

1. they are similar sets; and
2. they are equal in individual weight.

Formally:

$$\forall s1,s2(((s1 =_w s2) \,\&\, \text{similar\_reasonset}(s1, s2)) \rightarrow \text{Equal}(s1, s2))$$

13.3. Comparative reasoning about sets of contributive reasons

There may be other than logical grounds on which a set is stronger than, weaker than, or equal to another set. In fact, the determination which of two sets is stronger or weaker than the other will most often be just a matter of decision making. But also then the following relations should hold:

$$\forall s1,s2(\text{Stronger}(s1,s2) \equiv \sim\text{Weaker}(s1,s2))$$
$$\forall s1,s2(\text{Stronger}(s1,s2) \equiv \sim\text{Equal}(s1,s2))$$
$$\forall s1,s2(\text{Weaker}(s1,s2) \equiv \sim\text{Equal}(s1,s2))$$

There are more ways to reason about the relative strength of sets of contributive reasons, based on the transitivity of the stronger than, equal to, and weaker than relations.

If one set of reasons is *on logical grounds* stronger than another set, and this other set is against *on logical grounds* stronger than a third set, the first set is on logical grounds stronger than the third set. The same transitivity holds for the equal to and weaker than relations, *to the extent that they exist on logical grounds* specified in the previous section.

However, the logical characterisations of these relations as given in the previous section are not definitions. A reason set may be stronger than, equal to, or weaker than another reason set, just because it was decided to be so. As a consequence, the relations stronger/2, equal/2 and weaker/2 cannot be assumed to be transitive.

Nevertheless it is reasonable to assume that some weaker form of transitivity holds. If set 1 is stronger than set 2 and set 2 is stronger than set 3, then normally set 1 will be stronger than set 3. If set 1 is equal to set 2 and set 2 is equal to set 3, then normally set 1 will be equal to set 3. And, finally If set 1 is weaker than set 2 and set 2 is weaker than set 3, then normally set 1 will be weaker than set 3. This *weak transitivity* can be expressed in terms of abstract reasons as follows:

```
Ar((*stronger(a,b) & stronger(b,c)), *stronger(a,c))
Ar((*equal(a,b) & equal(b,c)), * equal(a,c))
Ar((*weaker(a,b) & weaker(b,c)), *weaker(a,c))
```

Moreover, if a set of contributive reasons *A* is stronger than set *B* and if set *B* is equal to set *C*, then *A* will normally be stronger than *C*:

```
Ar((*stronger(a,b) & equal(b,c)), *stronger(a,c))
```

If a set of contributive reasons *A* is equal to set *B* and if set *B* is stronger than set *C*, then *A* will normally be stronger than *C*:

```
Ar((*equal(a,b) & stronger(b,c)), *stronger(a,c))
```

If a set of contributive reasons *A* is weaker than set *B* and if set *B* is equal to set *C*, then *A* will normally be weaker than *C*:

```
Ar((*weaker(a,b) & equal(b,c)), *weaker(a,c))
```

If a set of contributive reasons *A* is equal to set *B* and if set *B* is weaker than set *C*, then *A* will normally be weaker than *C*:

```
Ar((*equal(a,b) & weaker(b,c)), *weaker(a,c))
```

13.4. Comparing alternatives

In the qualitative comparison of two alternatives, four sets of reasons are involved:

- the reasons pleading for the first alternative;
- the reasons pleading against the first alternative;
- the reasons pleading for the second alternative;
- the reasons pleading against the second alternative.

In principle, the first alternative is better than the second alternative, if either one of the following situations occurs:

1. The first alternative is stronger in pro-reasons than the second, while it is equal or weaker in the con-reasons.
2. The first alternative is weaker in con-reasons than the second, while it is equal in the pro-reasons.
3. The second alternative is weaker in pro-reasons than the first, while it is equal or stronger in the con-reasons.
4. The second alternative is stronger in con-reasons than the first, while it is equal in the pro-reasons.

Formally[26]:

```
Ar(
   (*stronger(r+(a1), r+(a2)) &
      weaker(r-(a1), r-(a2)) ∨ equal(r-(a1), r-(a2))) ∨
   (*weaker(r-(a1), r-(a2)) & equal(r+(a1), r+(a2))) ∨
   (*weaker(r+(a2), r+(a1)) &
      weaker(r-(a2), r-(a1)) ∨ equal(r-(a2), r-(a1))) ∨
   (*stronger(r-(a2), r-(a1)) & equal(r+(a1), r+(a2))),
      *better(a1, a2))
```

Similarly, the first alternative is worse than the second alternative, if either one of the following situations occurs:

1. The first alternative is weaker in pro-reasons than the second, while it is equal or stronger in the con-reasons.
2. The first alternative is stronger in con-reasons than the second, while it is equal in the pro-reasons.
3. The second alternative is stronger in pro-reasons than the first, while it is equal or weaker in the con-reasons.
4. The second alternative is weaker in con-reasons than the first, while it is equal in the pro-reasons.

Formally:

```
Ar(
   (*weaker(r+(a1), r+(a2)) &
      stronger(r-(a1), r-(a2)) ∨ equal(r-(a1), r-(a2))) ∨
   (*stronger(r-(a1), r-(a2)) & equal(r+(a1), r+(a2))) ∨
   (*stronger(r+(a2), r+(a1)) &
      stronger(r-(a2), r-(a1)) ∨ equal(r-(a2), r-(a1))) ∨
   (*weaker(r-(a2), r-(a1)) & equal(r+(a1), r+(a2))),
      *worse(a1, a2))
```

And the first alternative is equally good as the second alternative, if they are equal both in pro and con reasons:

```
Ar(*equal(r+(a1), r+(a2)) & equal(r-(a1), r-(a2)),
 *equally_good(a1,a2)
```

The relation `better/2` cannot be taken to be transitive, because the logical grounds on which this relation may be assumed do not guarantee their conclusion. But if alternative A is better than alternative B, while B is better than C, it is at least plausible that A is better than C. This weak transitivity can be expressed as follows:

```
Ar((*better(a,b) & better(b,c)), *better(a,c))
```

The same counts for the relations `worse/2` and `equally_good/2`:

```
Ar((*worse(a,b) & worse(b,c)), *worse(a,c))
Ar((*equally_good(a,b) & equally_good(b,c)),
 *equally_good(a,c))
```

13.5. Application of the formalisation

To illustrate the formalisation described above, I will formalize the example about the caustic soda case and the yew case where QCR is applied to case based reasoning. I will use the follow abbreviations:

---

[26] That the indicated relation only holds in principle is formalized by specifying the connection in terms of an abstract *contributive* reason.

Ds: defendant created a dangerous situation to which plaintiff fell victim
Ec: it was easy and cheap to avoid the danger
Dh: potential damages were high
Na: defendant was not aware that he created a danger
N: defendant was negligent

Csds: defendant created a dangerous situation to which plaintiff fell victim in the caustic soda case
Csec: it was easy and cheap to avoid the danger in the caustic soda case
Csdh: potential damages were high in the caustic soda case
Csna: defendant was not aware that he created a danger in the caustic soda case
Csn: defendant was negligent in the caustic soda case

Yds: defendant created a dangerous situation to which plaintiff fell victim in the yew case
Yec: it was easy and cheap to avoid the danger in the yew case
Ydh: the potential damages were high in the yew case
Yna: defendant was not aware that he created a danger in the yew case
Yn: defendant was negligent in the yew case

The following premises are assumed:

Csds & Csec & Csdh & Csna & Csn

Yds & Yec & Ydh & Yna

Ar(*ds, *n) & Ar(*ec, *n) & Ar(*dh, *n) & Ar(*na, *~n)

$\exists i$(Instantiates(*csds, *ds, i))
$\exists i$(Instantiates(*yds, *ds, i))
$\exists i$(Instantiates(*csec, Ec, i))
$\exists i$(Instantiates(*yec, *ec, i))
$\exists i$(Instantiates(*csdh, *dh, i))
$\exists i$(Instantiates(*ydh, *dh, i))
$\exists i$(Instantiates(*csna, *na, i))
$\exists i$(Instantiates(*yna, *na, i))
$\exists i$(Instantiates(*csn, *n, i))
$\exists i$(Instantiates(*yn, *n, i))

Given these premises, it is possible to derive that:

r^+(*csn) = {*csds, *csec, *csdh}
r^-(*csn) = {*cna}

r^+(*yn) = {*yds, *yec, *ydh}
r^-(*yn) = {*yna}

similar_reasonsets(r^+(*csn), r^+(*yn))
similar_reasonsets(r^-(*csn), r^-(*yn))

r^+(*csn)$=_w$ r^+(*yn)
r^-(*csn)$=_w$ r^-(*yn)

From these last four sentences it follows that:

equal(r^+(*csn), r^+(*yn))
equal(r^-(*csn), r^-(*yn))

and

equally_good(*csn, *yn)

To draw the additional conclusion that the defendant in the yew case acted negligently, an additional premise is necessary. If in one case a particular decision was taken, and a similar decision in another

case would be at least as good (equally good, or even better), this is a reason why this similar decision should be taken in the other case[27]:

```
Ar(
   *decision(c1,*d1) & similar(*d1,*d2) &
     (equally_good(*d1,*d2) ∨ better(*d1,*d2)),
   *ob(decision(c2,*d2))
```

In the caustic soda case the decision was that defendant acted negligently:

```
Decision(caustic_soda_case, *csn)
```

Moreover, the (possible) decisions `*csn` and `*yn` both instantiate `*n` and are therefore similar.

As a consequence it follows that:

```
Cr(
   *decision(caustic_soda_case, *csn)& similar(*csn,*yn) &
     (equally_good(*csn,*yn) ∨ better(*csn,*yn)),
       *ob(decision(yew_case,*yn))
```

There are no contributive reasons why the decision in the yew case should not be `*yn`[28] :

r^-(*ob(decision(yew_case,*yn))= ∅

Therefore the reasons why the decision in the yew case should be to assume negligence outweigh the reasons against this conclusion:

r^+(*ob(decision(yew_case,*yn))> r^-(*ob(decision(yew_case,*yn))

It therefore follows that in the yew-case negligence should be assumed:

```
Ob(decision(yew_case,*n))
```

14. Other research on legal comparative reasoning

Comparative reasoning belongs to the field of decision theory. A general treatment of decision making with multiple objectives can be found in Keeney and Raiffa 1993. To my knowledge, the idea that specifically topics *in the field of legal reasoning* can be treated as a form of comparative reasoning has only been explored before in Hage 2000 and 2001. In these two papers I focussed on legal theory construction and presented what is essentially the present theory of QCR.

In their (1997), Gordon and Karacipilidis developed a formal model of argumentation which also deals with the qualitative comparison of alternatives. This Zeno model has actually two ways to argue about relative preference. One way is to make the statement that one alternative is better than another alternative the conclusion of an argument. This does not involve a special technique for comparative reasoning, and is therefore not relevant for my present purposes. The other way is the application of one of a number of standards by means of which alternatives can be evaluated in relation to their alternatives. These standards are based on the standards for the evaluation of defeasible arguments, proposed in Freeman and Farley 1996. Gordon and Karacapilidis leave the question which standard should be applied to which alternative open for future research. The standards are:

Scintilla of evidence:
There is at least one valid reason pleading for this alternative. (Actually this is not a case of comparative reasoning, because the other alternatives play no role.)

Preponderance of the evidence:
The reasons pleading for this alternative outweigh the reasons against it. (This is not a real case of comparative reasoning either, for the same reason.)

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[27] In the formula below, the function expression `ob/1` (*ought to be*) is used to express that the decision in the second case *should be* `*d2`.

[28] The reason that defendant was not aware that he created a danger was already taken into account in the case comparison and is therefore not taken into consideration anymore (excluded; see section 12.4).

No better alternative:
It is consistent with the data that the reasons pleading for the alternative together outweigh the reasons for all the other alternatives. In this connection con-reasons are assigned a negative weight. This standard presupposes that it is somehow possible to sum the reasons, because otherwise the weights of the reasons could not be combined, and the sums of the weights not be compared. The authors do not mention how this summing should take place; they only write that they are interested in qualitative, not in quantitative reasoning. Presumably this implies that relative values of the summed weights should be supplied as premises. For instance, it should explicitly be given that the summed weight of the reasons that a Volvo is a safe car in combination that there is a dealer nearby is bigger than the weight of the reason that a Porsche is good for one's image.

Best choice:
The reasons pleading for the alternative together outweigh the reasons for all the alternatives. In this connection con-reasons are assigned a negative weight. This makes the same presupposition about the summing of reasons as the previous standard.

Beyond a reasonable doubt:
There are no reasons against this alternative and no reasons for the other alternatives.

From these five standards, only the last three deal with real qualitative reasoning; the other two standards do not mention other alternatives than the one that is evaluated. From these last three, the last standard concerns an extreme case. The standards 'no better alternative' and 'best choice' presuppose that the relevant weights and sums of weights are provided as premises (if the weights were computed, the Zeno approach would not exemplify qualitative reasoning) and consequently provide less guidance for comparative qualitative reasoning than the model proposed in this paper.

An interesting feature of the Zeno system is that it allows for the comparison of sets of alternatives. Suppose that you are looking for a car, which you intend to take on a holiday for which the destination must also be chosen. There are two possible cars, a Volvo and a Ferrari, and two possible destinations, Italy and Sweden. The Zeno system allows the comparison of combinations of alternatives. For instance, it would be possible to compare buying a Volvo and go on holiday to Sweden with buying a Ferrari and spending the holiday in Italy. This is not possible in the version of QCR proposed here.

15. Conclusion

In this paper I hope to have done three things:

1. Expose a general theory about qualitative comparative reasoning in terms of sets of reasons that plead for and against alternatives.
2. Show how this theory underlies already existing work on case comparison and can be used in connection with legal theory construction and reasoning about legal proof.
3. Reconceptualize and expand Reason-based Logic to make it suitable for dealing with the presented theory about qualitative comparative reasoning.

Moreover, I hope to have made clear that the proposed theory of QCR has a broader range of application than might seem at first sight, because

- of the weak transitivity of a number of crucial relations between alternatives and reason sets, and
- the relations stronger, weaker and equal as they are defined between reason sets, are open in the sense that one set may be stronger than another one on non-logical grounds, for instance because it was decided to be stronger.

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